Chapter 5

Electric Potential

Assoc. prof. / Amr Hessein

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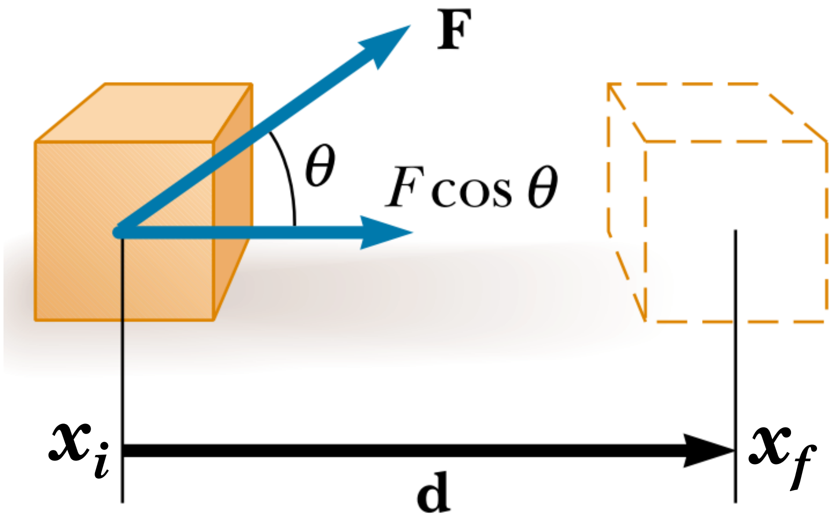
1. Electric work done (w)

- The **work (W)** done on an object by a **force (F)** is:

$$W = \int_{x_i}^{x_f} \vec{F} \cdot d\vec{x} = \int_{x_i}^{x_f} F dx \cos\theta$$



Remarks

- $if\ \theta = 90° \rightarrow W = 0$, The work done by a force on a moving object is zero when the force applied is perpendicular to the object's displacement.
- **Work is a scalar quantity because work is an energy transfer**;
  - ✓ If energy is transferred to the system (object), **W is positive**;
  - ✓ If energy is transferred from the system, **W is negative**.

1. Electric work done (w)

➢ **Conservative Force القوة المحافظة**

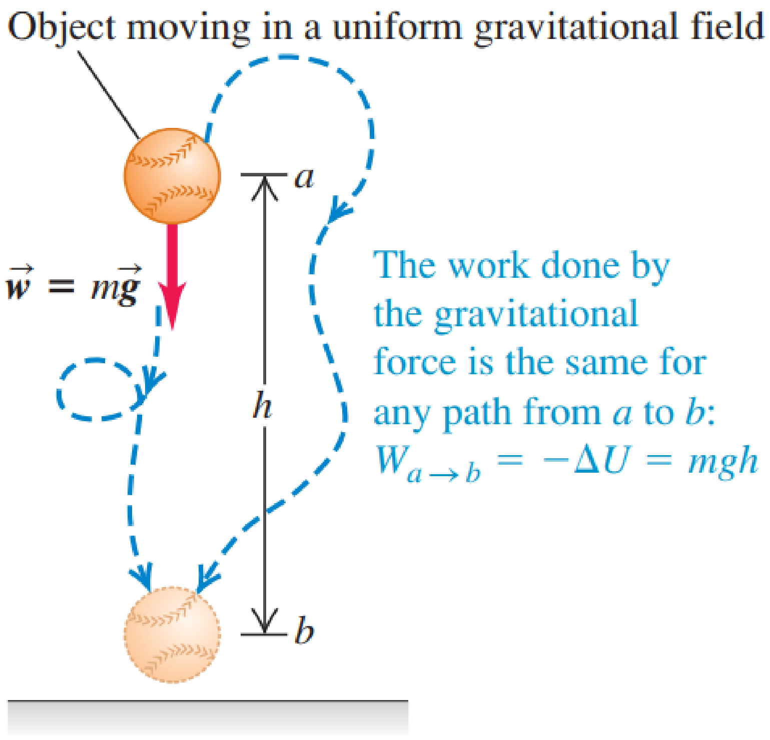
1. The work done between any two points is **independent of the path taken by the particle**.
2. The work done through any closed path is **zero.**

$$\oint \vec{F} \cdot d\vec{x} = zero$$

➢ Examples on a **conservative force** are:

✓ **Gravitational force**

✓ **Electrostatic force**



➢ The work done by conservative force can always be expressed in terms of **change in Potential Energy (ΔU)**

$$W_{ab} = -\Delta U$$

1. Electric work done (w)

➢ **Conservative Force القوة المحافظة**

$$W_{ab} = -\Delta U$$

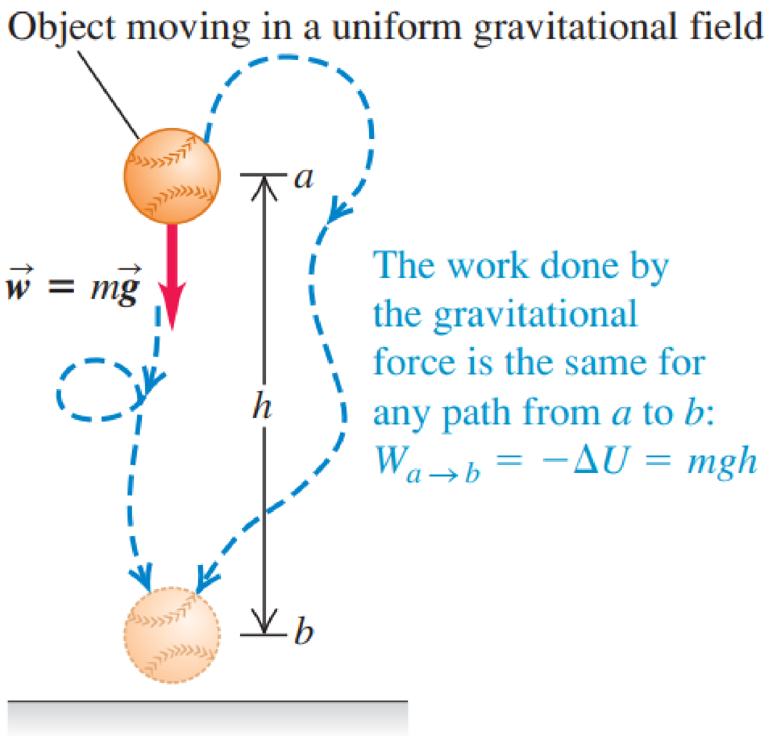
$$W_{a\to b} = U_a - U_b = -(U_b - U_a) = -\Delta U$$

(i) $W_{a\to b} = +ve, \rightarrow \Delta U = -ve$

i.e. the potential energy ***decreases***

(ii) $W_{a\to b} = -ve, \rightarrow \Delta U = +ve$

i.e. the potential energy ***increases***



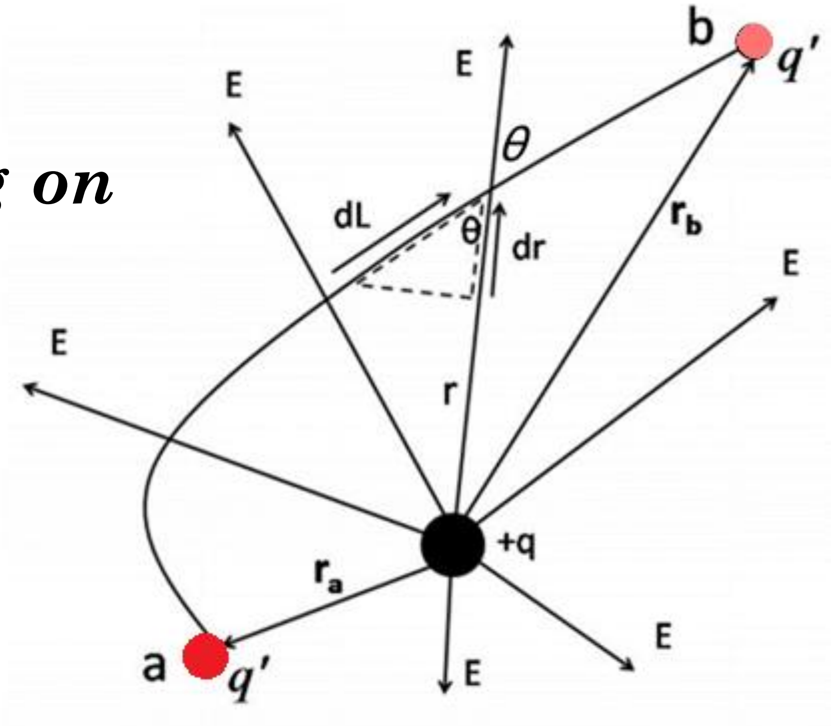
1. Electric work done (w)

- Now, consider a **central positive charge q**, produce electric field **E** everywhere surround it and has infinite range:

$$E = K\frac{q}{r^2}$$

- ***W_{ab} is done by electrostatic force ($q'E$) exerting on q' by the field E through the path $a \rightarrow b$.***

$$W_{ab} = Kqq'\left[\frac{1}{r_a} - \frac{1}{r_b}\right] \quad \textit{Joule}$$



- It is clear that, the work don is <u>**independent on the shape of the path**</u> joined a → b but only on the radial distances of initial and final positions (***r_a & r_b***).

  - **So, the electric force is a Conservative Force**

1. Electric work done (w)

$$\therefore W_{ab} = Kqq' \left[\frac{1}{r_a} - \frac{1}{r_b}\right]$$ *Joule*

➢ For similar charges, the work may positive or negative according the magnitudes of (r):

(1) *If* $r_a < r_b$

$W_{ab} = +ve$ *The work is done by the field, and q' moves away from q*

(2) *If* $r_a > r_b$

$W_{ab} = -ve$ *The work is done on the field , and q' moves towards from q*

(3) *If* $r_a = r_b$,

$W_{ab} = 0$ *when q' moves on surface of the sphere.*

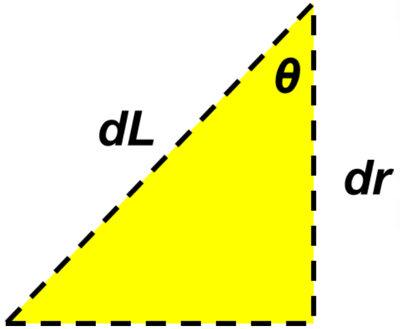
1. Electric work done (w)

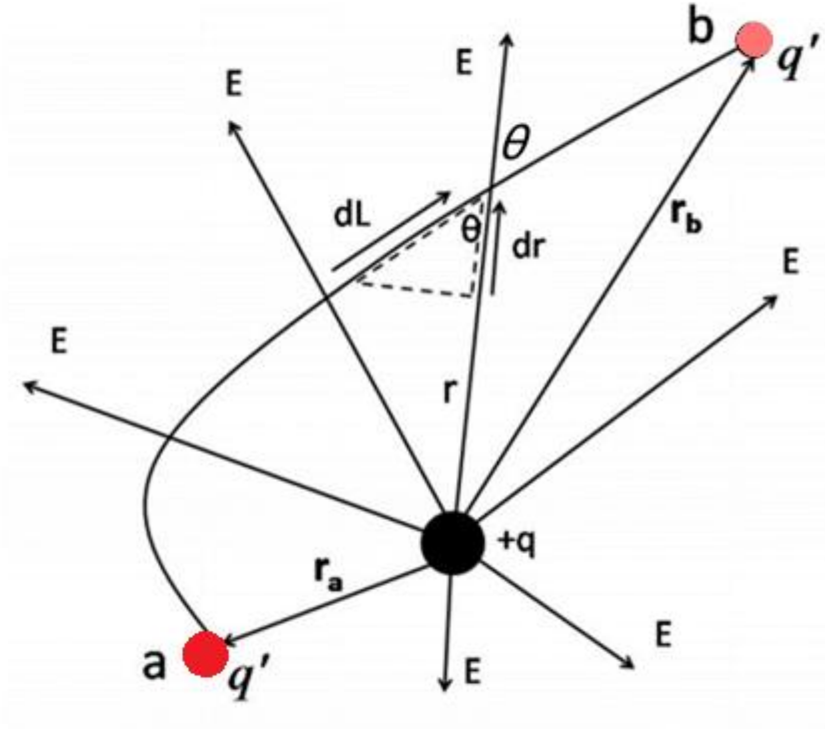
➢ ***Prove that the electrostatic force is a conservative force?***

$$dW = \vec{F} \cdot \overrightarrow{dL} = F dL \cos\theta$$

$$dr = dL\cos\theta$$

$$dW = F dr$$





➢ ***the total work don W_{ab} is:***

$$W_{ab} = \int_{r_a}^{r_b} F dr \ = q' \int_{r_a}^{r_b} E dr$$

$$E = K\frac{q}{r^2}$$

$$W_{ab} = Kqq' \int_{r_a}^{r_b} \frac{dr}{r^2} \ = Kqq' \left[-\frac{1}{r}\right]_{r_a}^{r_b}$$

$$\therefore W_{ab} = Kqq'\left[\frac{1}{r_a} - \frac{1}{r_b}\right]$$

##

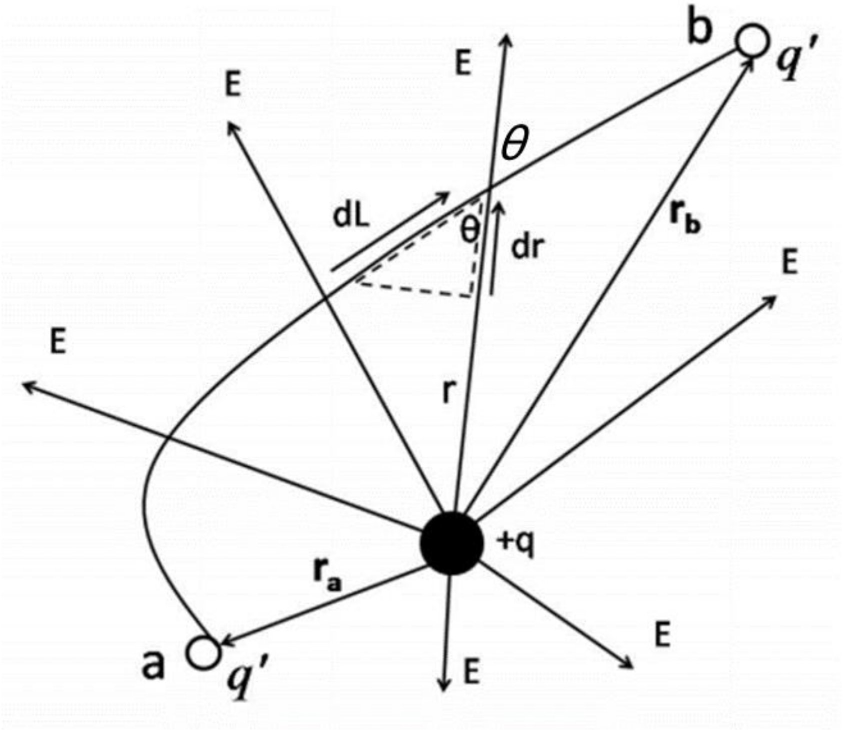
2. Electric potential energy (E_P) طاقة الوضع الكهربي

- **Since the electric force is a Conservative Force**

$$\therefore W_{ab} = -\Delta E_p$$

$$Kqq'\left[\frac{1}{r_a} - \frac{1}{r_b}\right] = (E_p)_a - (E_p)_b$$

$$\therefore (E_p)_a = K\frac{qq'}{r_a} \quad \& \quad (E_p)_b = K\frac{qq'}{r_b}$$



- ***In general,***

$$E_p = K\frac{qq'}{r}$$ ***Joule***

2. Electric potential energy (E_P)

$$E_p = K\frac{qq'}{r} \quad \textit{Joule}$$

Remarks

1. The potential energy E_p is a **shared property** of the two charges.

2. The electric potential energy is a **scalar quantity** is **positive for similar charges** and **negative for different charges**.

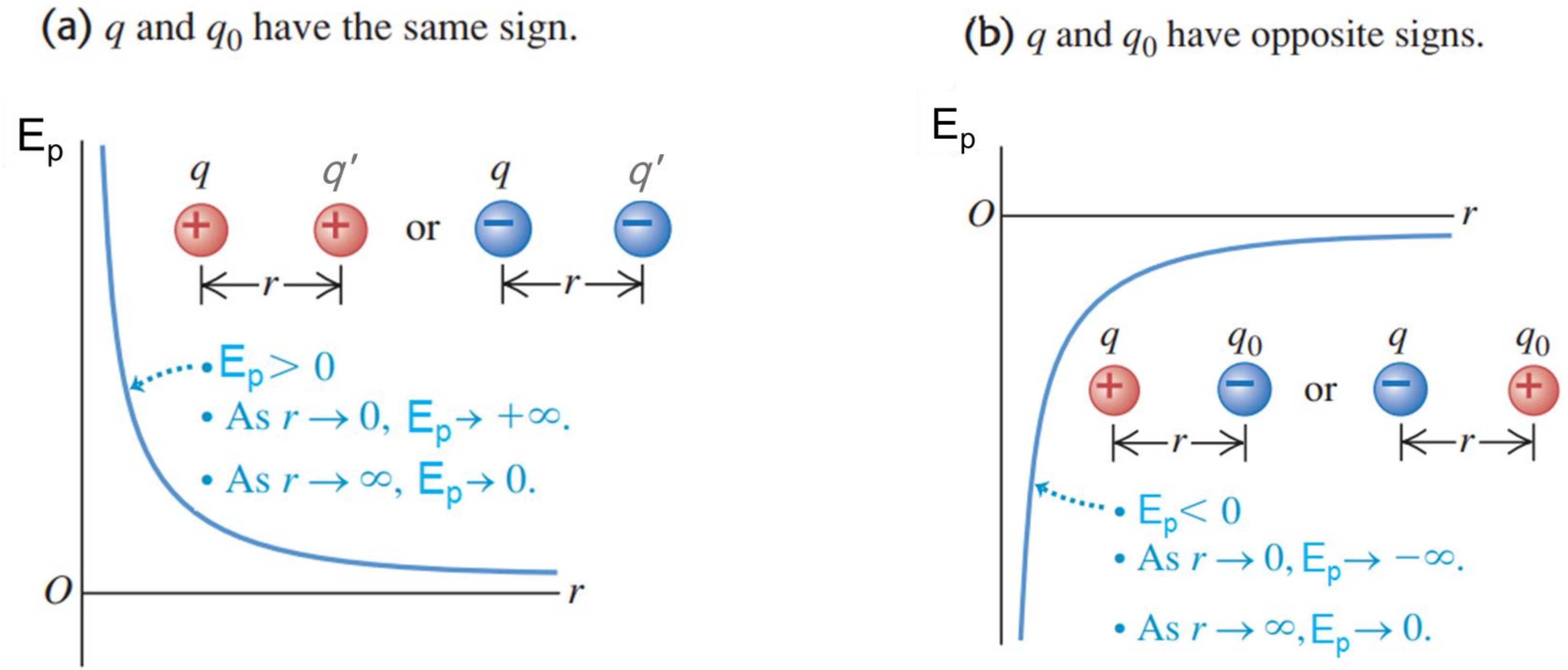
3. The potential energy is always defined relative to **some reference point** where $E_p = 0$

$$if\ r = \infty, \quad \rightarrow \quad E_p = zero$$

2. Electric potential energy (E_P)

$$E_p = K\frac{qq'}{r}$$

Joule

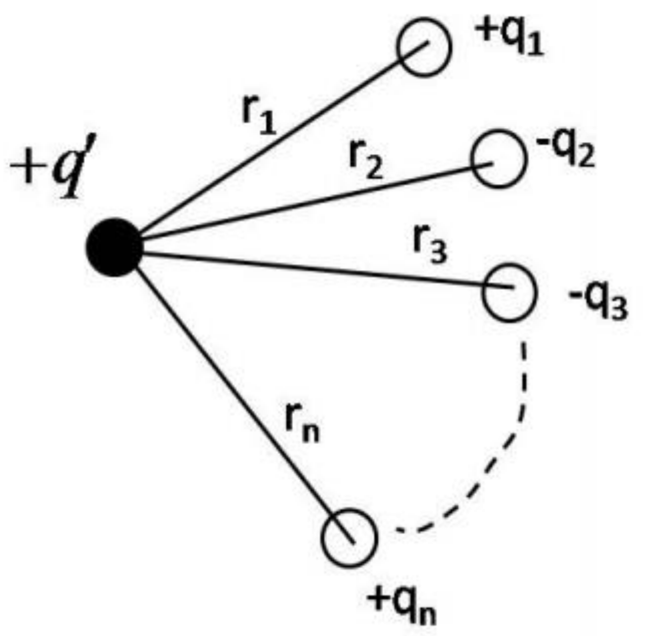


2. Electric potential energy (E_P)

- **If there is a system of n-charges, the total electric potential energy of q' is the algebraic sum of each potential energy of q' with each individual charge in the system**

$$E_p = q'\sum_{i=1}^{n} K\frac{q_i}{r_i}$$

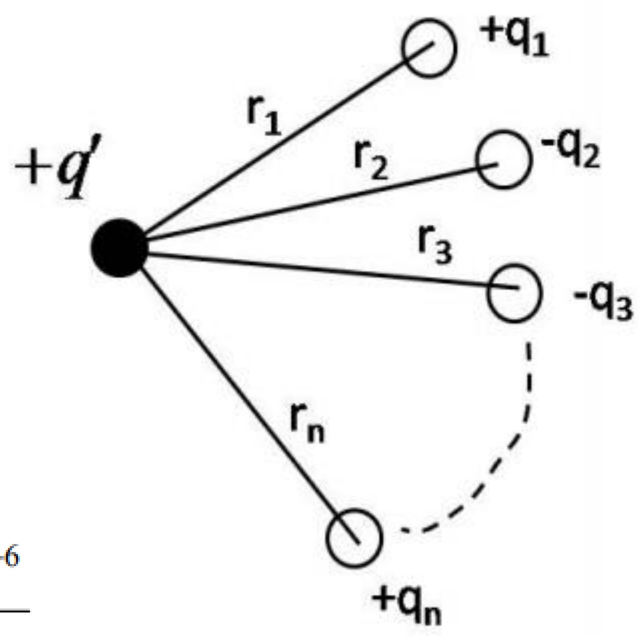
$$E_p = (+)K\frac{q_1 q'}{r_1} + (-)K\frac{q_2 q'}{r_2} + (-)K\frac{q_3 q'}{r_2} + \ldots\ldots\ldots + K\frac{q_n q'}{r_n}$$



Example 2: in fig. 2 let test charge $q' = +6\mu C$ is surrounded by three charges where $q_1 = +1\mu C$, $q_2 = -2\mu C$ and $q_3 = -3\mu c$ at distances 40, 70 and 90 cm from q' respectively. Calculate the electric potential energy on q'.

Solution

$$E_p = q' \sum_{i=1}^{n} K \frac{q_i}{r_i}$$



$$E_p = K\frac{q_1 q'}{r_1} + K\frac{q_2 q'}{r_2} + K\frac{q_3 q'}{r_2}$$

$$E_p = 9\times10^9 \frac{1\times10^{-6}\times6\times10^{-6}}{0.4m} - 9\times10^9 \frac{2\times10^{-6}\times6\times10^{-6}}{0.7m} - 9\times10^9 \frac{3\times10^{-6}\times6\times10^{-6}}{0.9m}$$

$$E_p = 0.135 - 0.154 - 0.18 = \underline{-0.2\, joule}$$

3. Electric potential (V) الجهد الكهربي

➢ Electric Potential (V)

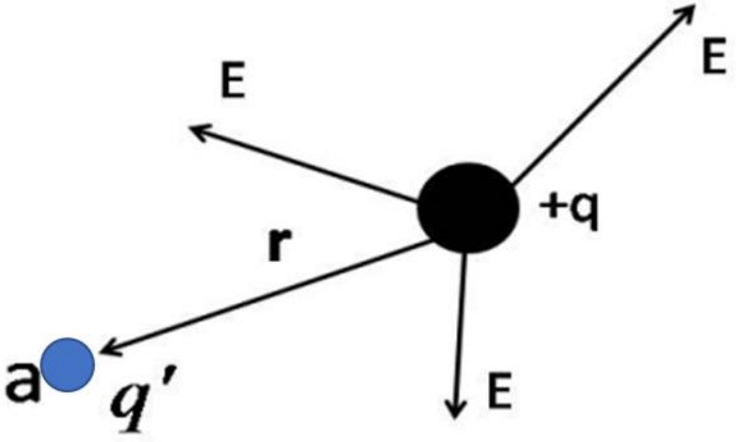
It is the potential energy per unit positive charge

$$E_p = K\frac{qq'}{r} \quad \Rightarrow \quad V = \frac{E_p}{q'} = \frac{K\frac{qq'}{r}}{q'}$$

➢ So, the electric potential at point (a) distant (r) from q is:

$$\boldsymbol{V = K\frac{q}{r}}$$

$$V = \frac{Joule}{Coulomb} \equiv Volt\ (V)$$



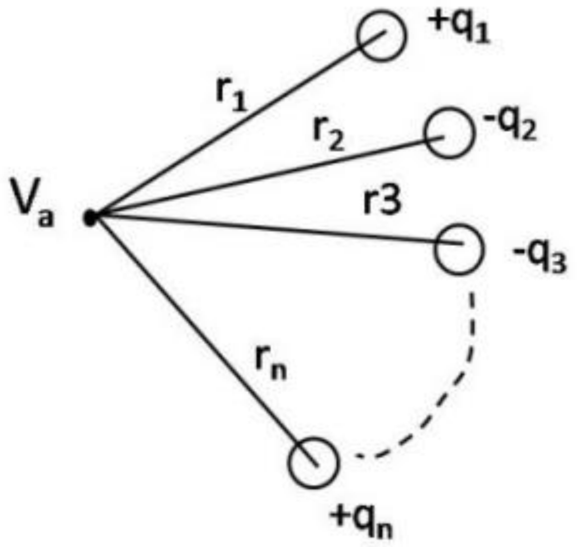
➢ The electric potential is <u>scalar quantity</u> positive for positive q and negative for negative q.

3. Electric potential (V)

➢ **If there is a system of n-charges, the electric potential at a point (a) is the algebraic sum of all potentials of that charges make at this point**

$$V_a = V_1 + V_2 + \ldots\ldots\ldots V_n$$

$$V_a = (+)K\frac{q_1}{r_1} + (-)K\frac{q_2}{r_2} + (-)K\frac{q_3}{r_2} + \ldots\ldots\ldots + K\frac{q_n}{r_n}$$



In general,

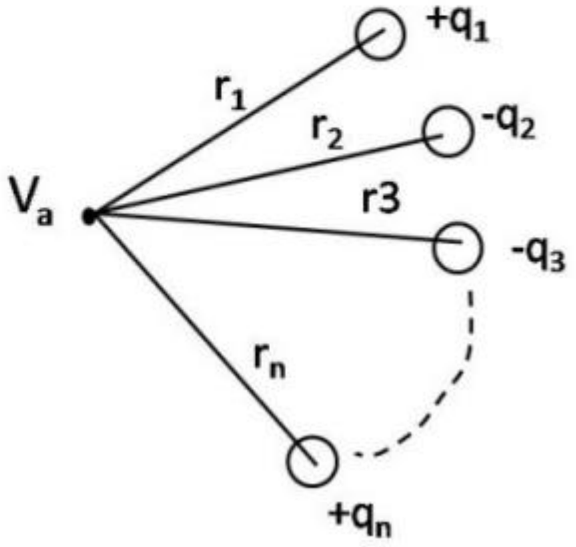
$$V_a = \sum_{i=1}^{n} K\frac{q_i}{r_i}$$

➢ **If any charge q’ is placed at point (a), it will have an electric potential energy (E_p) equals:**

$$E_p = q'V_a$$

Example 3: Let point a is surrounded by three charges where $q_1 = +1\mu C$, $q_2 = -2\mu C$ and $q_3 = -3\mu C$ at distances 40, 70 and 90 cm from it respectively. Calculate the electric potential at point a. If charge $+6\mu C$ is placed at point a calculate the subjected electric potential energy

Solution



$$V_a = K\frac{q_1}{r_1} + K\frac{q_2}{r_2} + K\frac{q_3}{r_2}$$

$$V_a = 9\times10^9 \frac{1\times10^{-6}}{0.4m} - 9\times10^9 \frac{2\times10^{-6}}{0.7m} - 9\times10^9 \frac{3\times10^{-6}}{0.9m}$$

$$V_a = 22500V - 25714V - 30000V = \underline{-33214V}$$

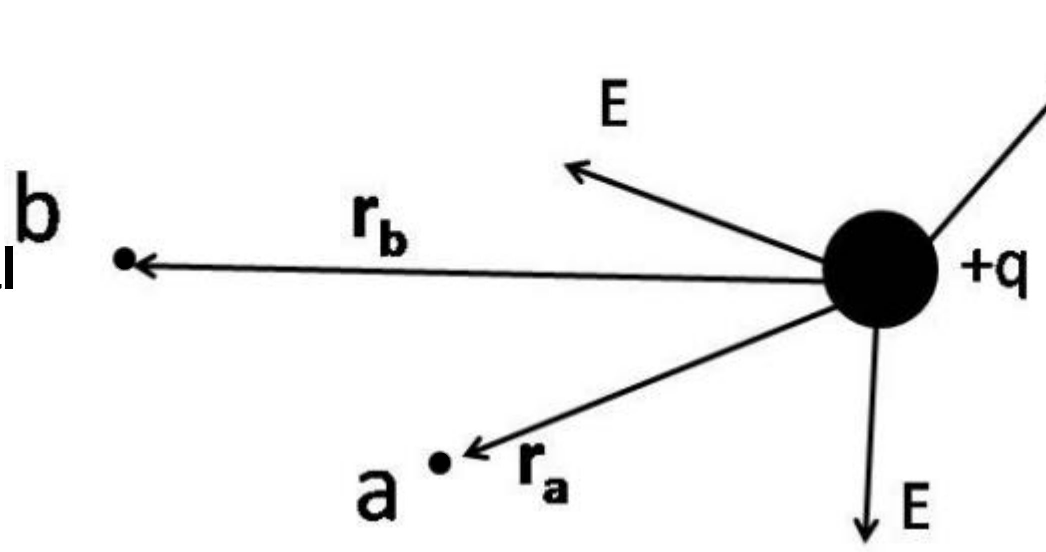
To calculate the potential energy E_p on charge $+6\mu C$ palced at a

$$E_p = q'V = +6\times10^{-6}C\times(-33214\ V) = \underline{-0.2\ Joule}$$

4. Electric potential difference (V_{ab})

- **There are different magnitudes of potential V because different distance r from the central charge q or distance from source of field**

$$V_a = K\frac{q}{r_a} \quad \text{and} \quad V_b = K\frac{q}{r_b}$$



- **The quantity (V_a - V_b) is called potential difference V_{ab} between points a and b:**

$$V_{ab} = V_a - V_b = K\frac{q}{r_a} - K\frac{q}{r_b}$$

- **If $r_a = r_b$ for spherical surface;**

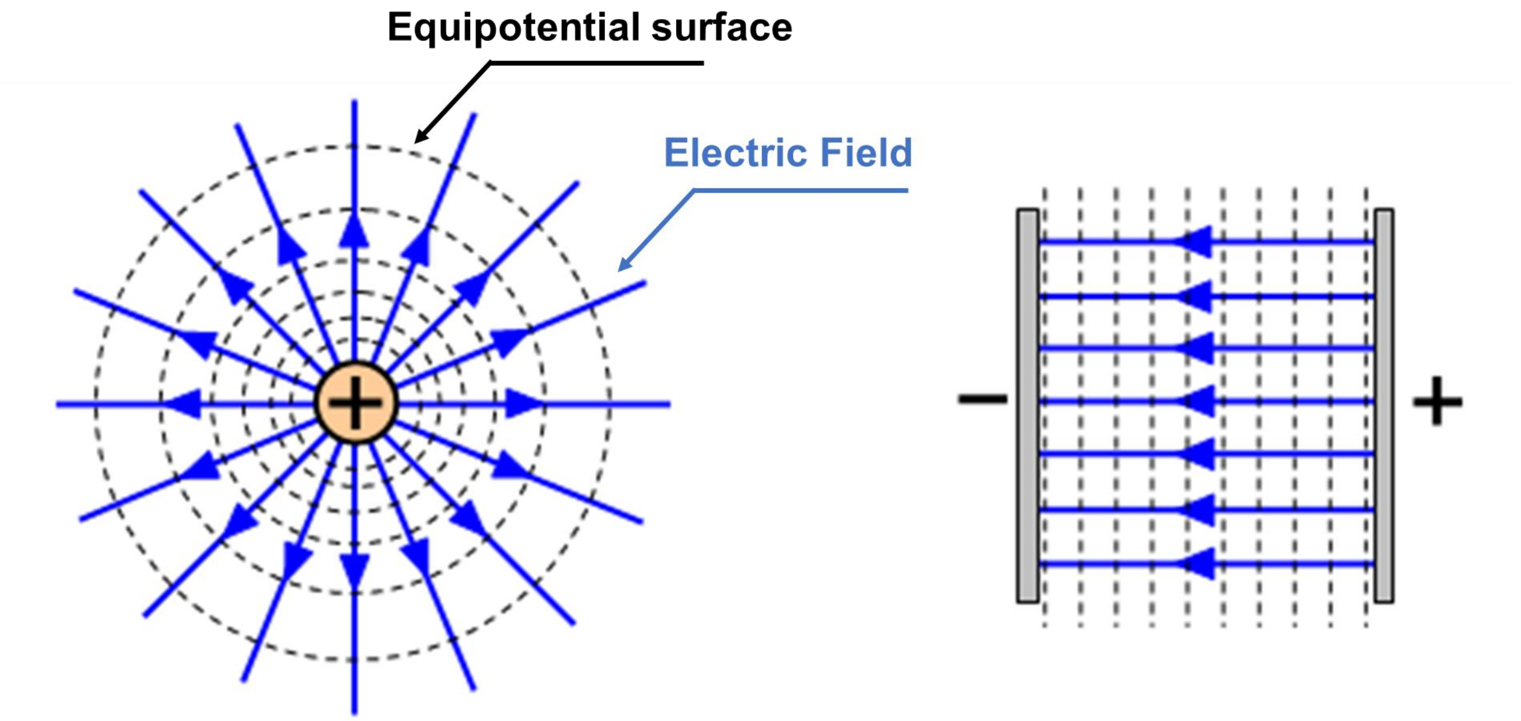
$$V_{ab} = V_a - V_b = 0$$

- **This surface is called Equipotential surface.**

4. Electric potential difference (V_{ab})

➢ **Equipotential Surface سطح متساوي الجهد الكهربي**

It is the surface at which the potential has the same value at all points on the surface.



➢ **No net work W is done on a charged particle by an electric field when the particle moves between two points on the same equipotential surface.**

5. Electric Energy (W_{ab})

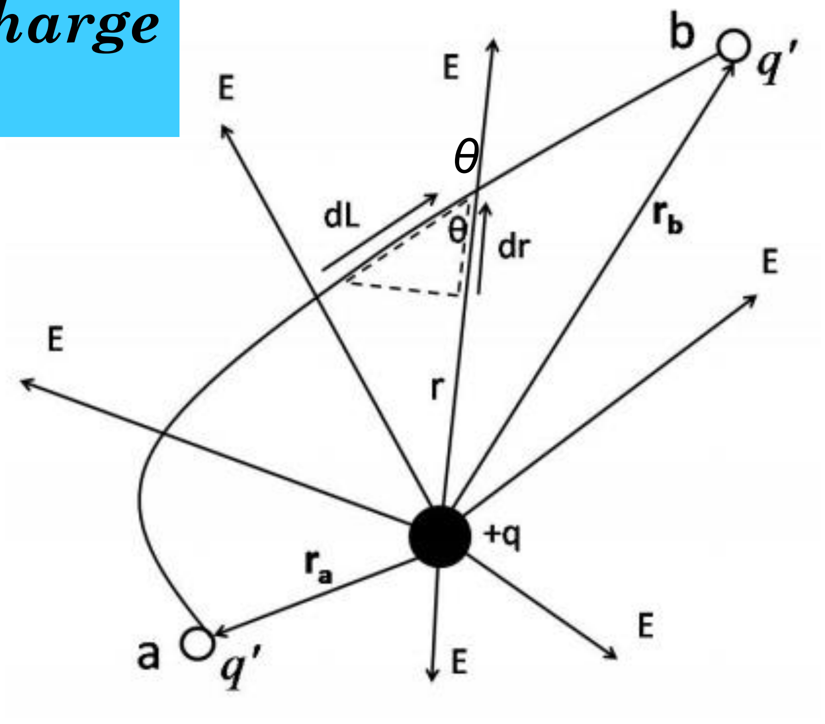
- **The electric potential difference (V_{ab}) between two points**

Is the quantity of work or energy gained per unit charge when transfer between them.

$$V_{ab} = \frac{W_{ab}}{q'}$$

- The electric energy (W_{ab}) is the quantity of energy responsible for motion of electric charge between two points a and b in an electric field.

$$W_{ab} = q'(V_a - V_b) = q'V_{ab}$$



- **If the electric energy is converted to kinetic energy:**

$$Kientic\ Energy\ (K.E) = W_{ab} = q'V_{ab} = q'(V_a - V_b)$$

5. Electric Energy (W_{ab})

- ***If*** $q' = e$ ***and*** $V_{ab} = 1$ ***Volt***

$$K.E = 1.6 \times 10^{-19} C \times 1\,V = 1.6 \times 10^{-19}\,Joule = 1\,eV$$

$$\boldsymbol{Electron - volt\ (eV) = 1.6 \times 10^{-19}\ Joule}$$

- **Electron-volt**

  is the quantity of kinetic energy gained by an electron when accelerated through a potential difference of one volt.

- eV usually used to measure the energy of small particles: **e.g. electrons, protons, ….**

- **The electric power (P) is the time rate of doing a work**

$$P = \frac{dW_{ab}}{dt}\ \text{Joules/s} \equiv \text{Watt (W)}$$

$$P = \frac{d(q'V_{ab})}{dt} = V_{ab}\frac{dq'}{dt} \Rightarrow \boldsymbol{P = V_{ab} I}$$

$I = \frac{dq'}{dt}$, ***is the electric current***

Example 4: A 20 gm of mass carry positive charge 50 μC transfer between two pints of potential difference 220 V. Calculate the (a) the gained kinetic energy (b) the work done per unit charge (c) velocity of transferring

Solution

(a) The gained kinetic energy

$$Kinetic\,energy\;\; K.E = q' V_{ab} = 50\times10^{-6}\times220 = \underline{0.011\,Joule}$$

(b) the work done per unit charge is the same potential difference $=$ $\underline{220\text{V}}$

(c) The velocity v is calculate from kinetic energy

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\times0.011\,J}{20\times10^{-3}\;Kg}} = \underline{1.04 m/s}$$

6. Potential and Electric Field

- There is a relation between electric field intensity and potential difference:

$$W_{ab} = q' \int_{r_a}^{r_b} E dr \quad \Rightarrow \quad \therefore \frac{W_{ab}}{q'} = \int_{r_a}^{r_b} E dr$$

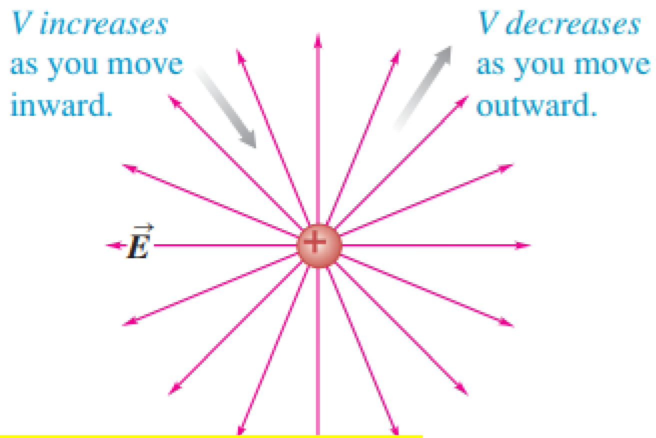
$$W_{ab} = q' V_{ab}$$

$$\boldsymbol{V_{ab} = \int_{r_a}^{r_b} E dr}$$

- The electric field can be found from the potential difference as:

$$\vec{E} = -\frac{dV}{dr}$$

$$V/m \equiv N/C$$



$\frac{dV}{dr} \rightarrow$ *is called* **the potential gradient**

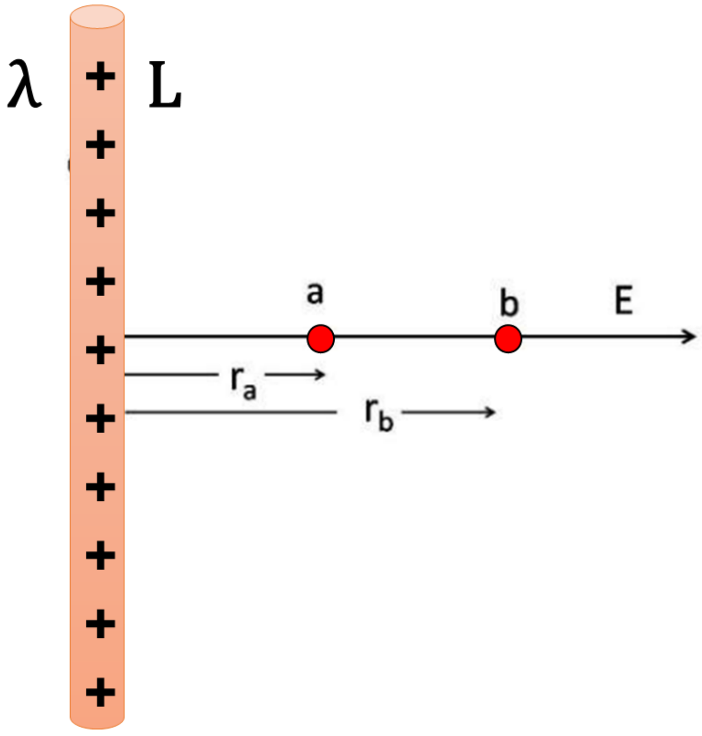
- **The negative (−ve) sign because the direction of E is usually in a direction of decreasing V with r**

❑ Potential difference in front of infinite charged filament

➢ ***Find the potential difference between two points (a & b) in front of an infinite charged filament?***

➢ ***the electric field intensity (E) due to charged infinite filaments is:***

$$E = \frac{2K\lambda}{r}$$



➢ ***So,***

$$V_{ab} = \int_{r_a}^{r_b} E dr = 2K\lambda \int_{r_a}^{r_b} \frac{dr}{r}$$

$$V_{ab} = 2K\lambda[\ln r]_{r_a}^{r_b} = 2K\lambda[\ln r_b - \ln r_a]$$

$$V_{ab} = 2K\lambda \ln\left(\frac{r_b}{r_a}\right)$$

➢ ***If*** $r_b > r_a \rightarrow V_{ab}$ ***=+ve or*** $V_a > V_b$***, and vice versa***

❑ Potential difference in front of infinite charged plane

- ***Find the potential difference between two points (a & b) in front of an infinite charged plane?***

$$V_{ab} = \int_{r_a}^{r_b} E dr$$

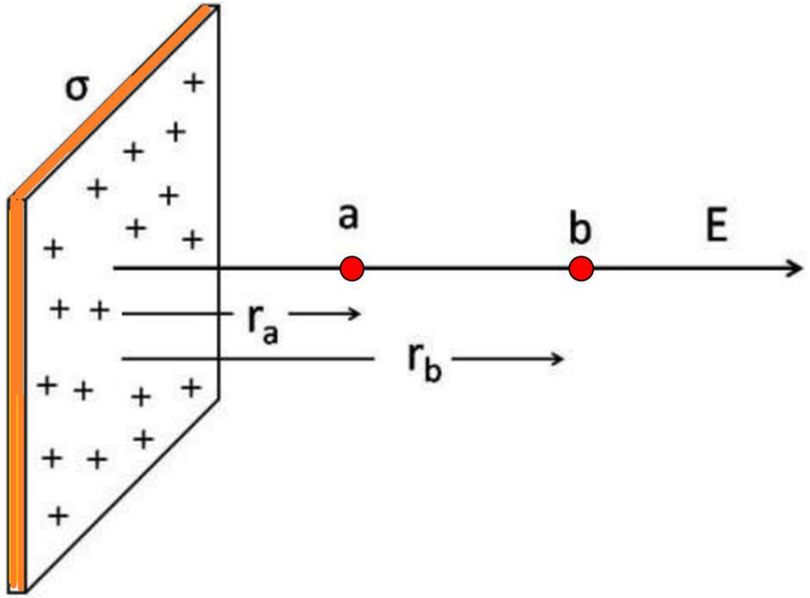
- ***The electric field of the plane is uniform and equals to E***

$$V_{ab} = E\int_{r_a}^{r_b} dr$$

$$V_{ab} = E(r_b - r_a) = Ed$$

$$\boldsymbol{V_{ab} = Ed}$$

$Where \rightarrow \ d = r_b - r_a$



$$E = \frac{\sigma}{2\varepsilon_o} \rightarrow for\ non - conducting\ plane$$

$$E = \frac{\sigma}{\varepsilon_o} \rightarrow for\ conducting\ plane$$

❑ Potential difference of charged sphere

➢ ***(A) If the charged sphere is conducting***

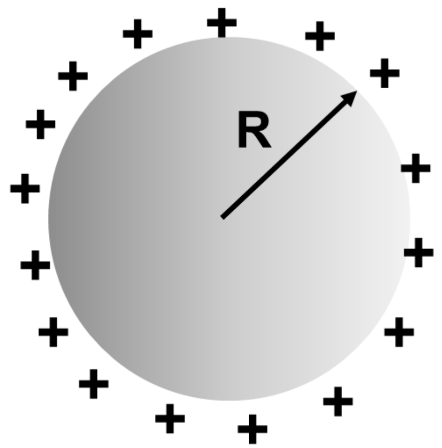
(i) If two points (a & b) are inside the sphere

$$E = 0 \rightarrow V_{ab} = 0$$

$$V_a = V_b$$

$$V_{inside} = V_{surface} = K\frac{q}{R}$$

Constant value



(ii) If the two points outside the sphere r>R

$$E = K\frac{q}{r^2}$$

$$V_{ab} = Kq\int_{r_a}^{r_b}\frac{dr}{r^2}$$

$$V_{ab} = Kq\left[\frac{1}{r_a} - \frac{1}{r_b}\right]$$

❑ Potential difference of charged sphere

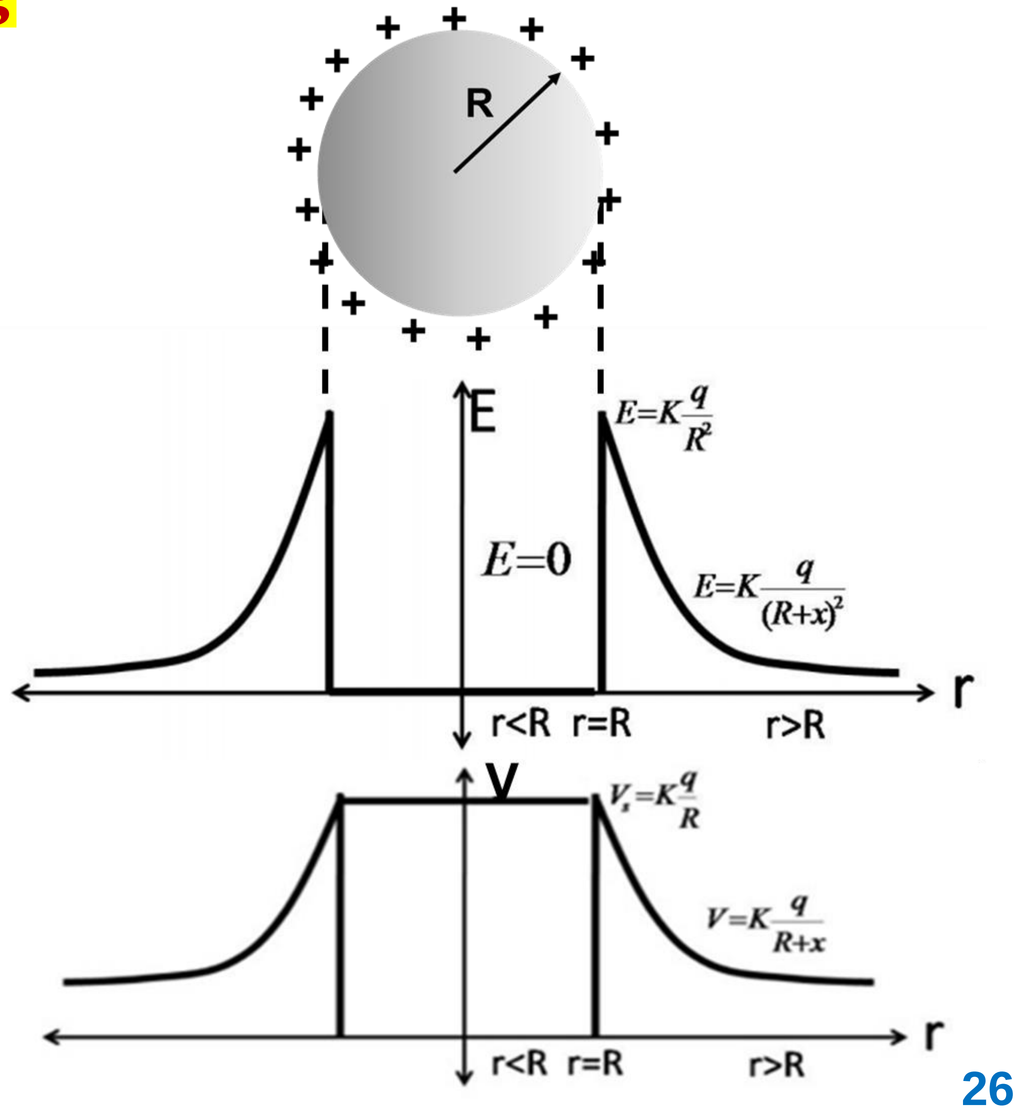
➢ ***(A) If the charged sphere is conducting***

(i) If two points (a & b) are inside the sphere

$$V_{inside} = V_{surface} = K\frac{q}{R}$$

(ii) If the two points outside the sphere r>R

$$V_{ab} = Kq\left[\frac{1}{r_a} - \frac{1}{r_b}\right]$$



❑ Potential difference of charged sphere

➢ ***(B) If the charged sphere is non-conducting***

(i) If two points (a & b) are inside the sphere

$$E = K\frac{qr}{R^3}$$

$$V_{ab} = K\frac{q}{R^3}\int_{r_a}^{r_b} r\,dr \Rightarrow V_{ab} = K\frac{q}{2R^3}\left(r_b^2 - r_a^2\right)$$

(ii) If the two points outside the sphere r>R

$$E = K\frac{q}{r^2}$$

$$V_{ab} = Kq\left[\frac{1}{r_a} - \frac{1}{r_b}\right]$$

❑ Potential difference of charged sphere

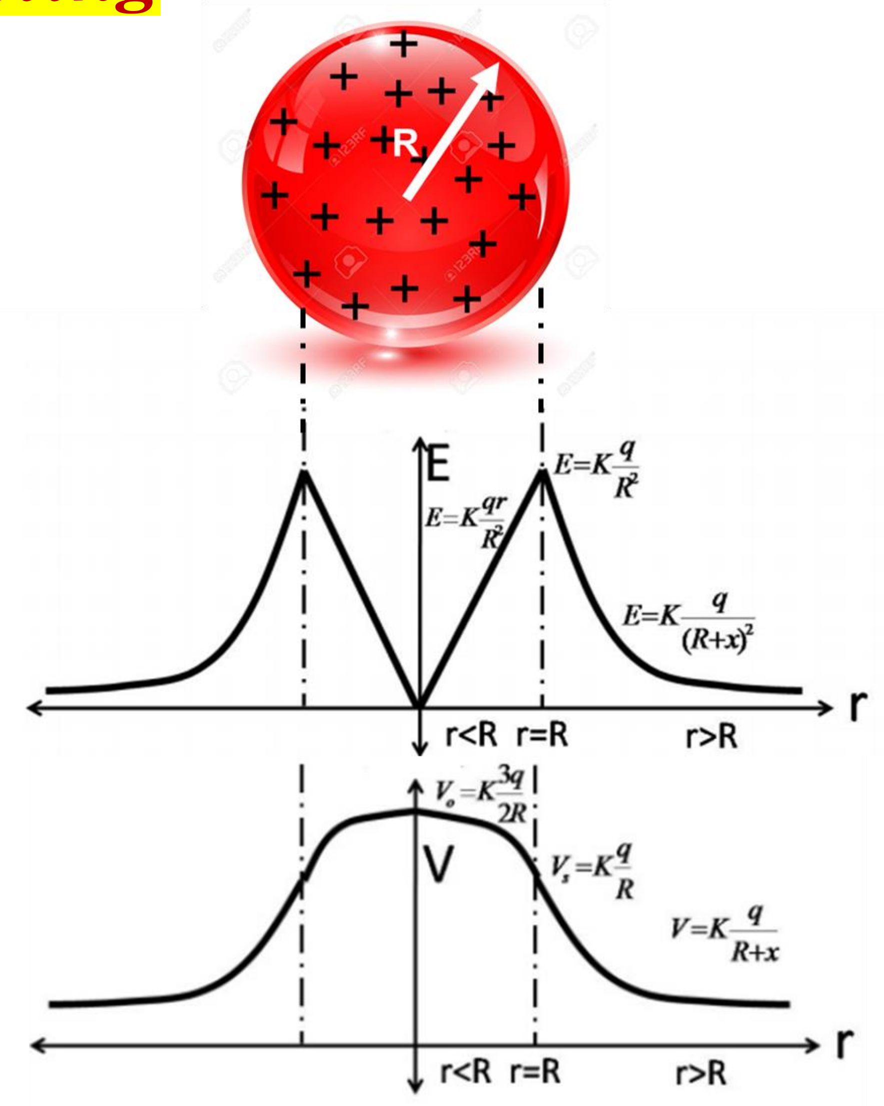
➢ ***(B) If the charged sphere is non-conducting***

(i) If two points (a & b) are inside the sphere

$$V_{ab} = K\frac{q}{2R^3}\left(r_b^2 - r_a^2\right)$$

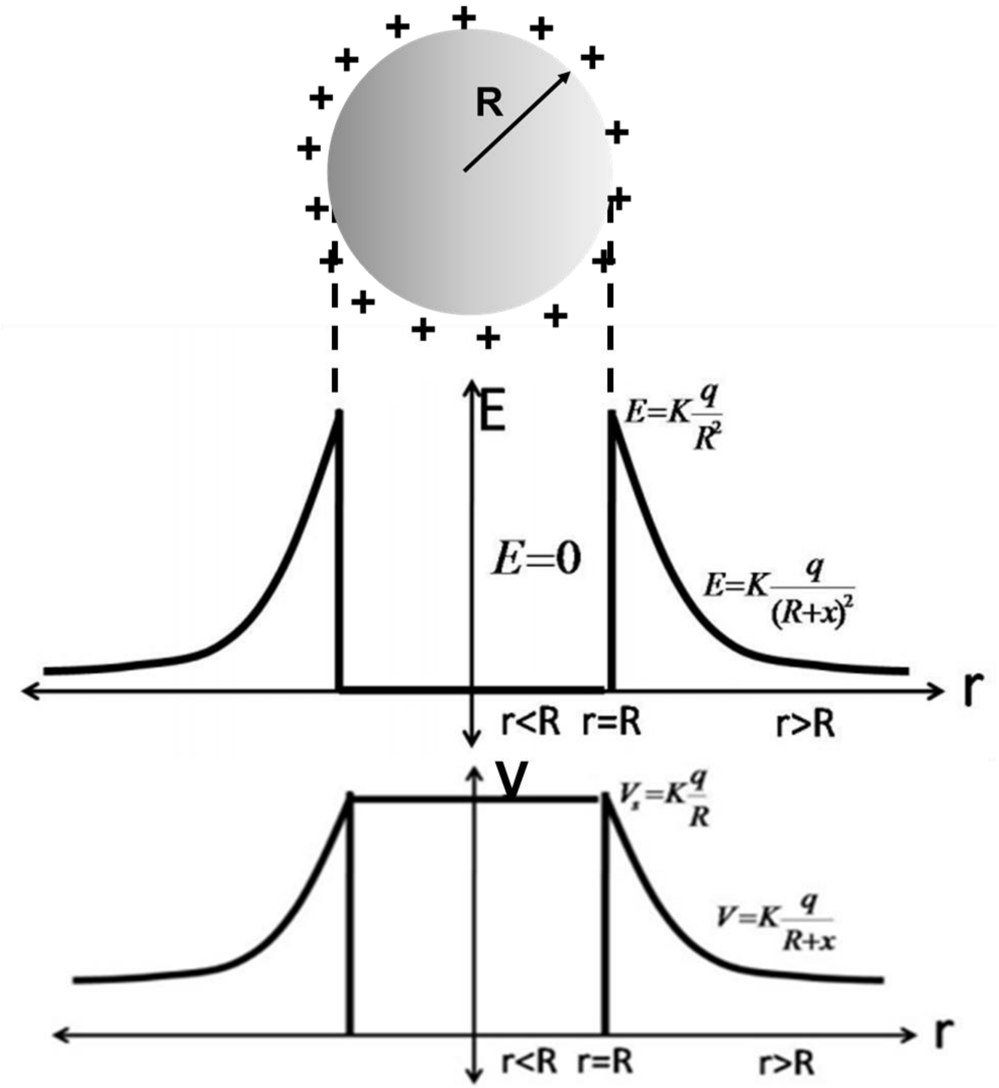
(ii) If the two points outside the sphere r>R

$$V_{ab} = Kq\left[\frac{1}{r_a} - \frac{1}{r_b}\right]$$

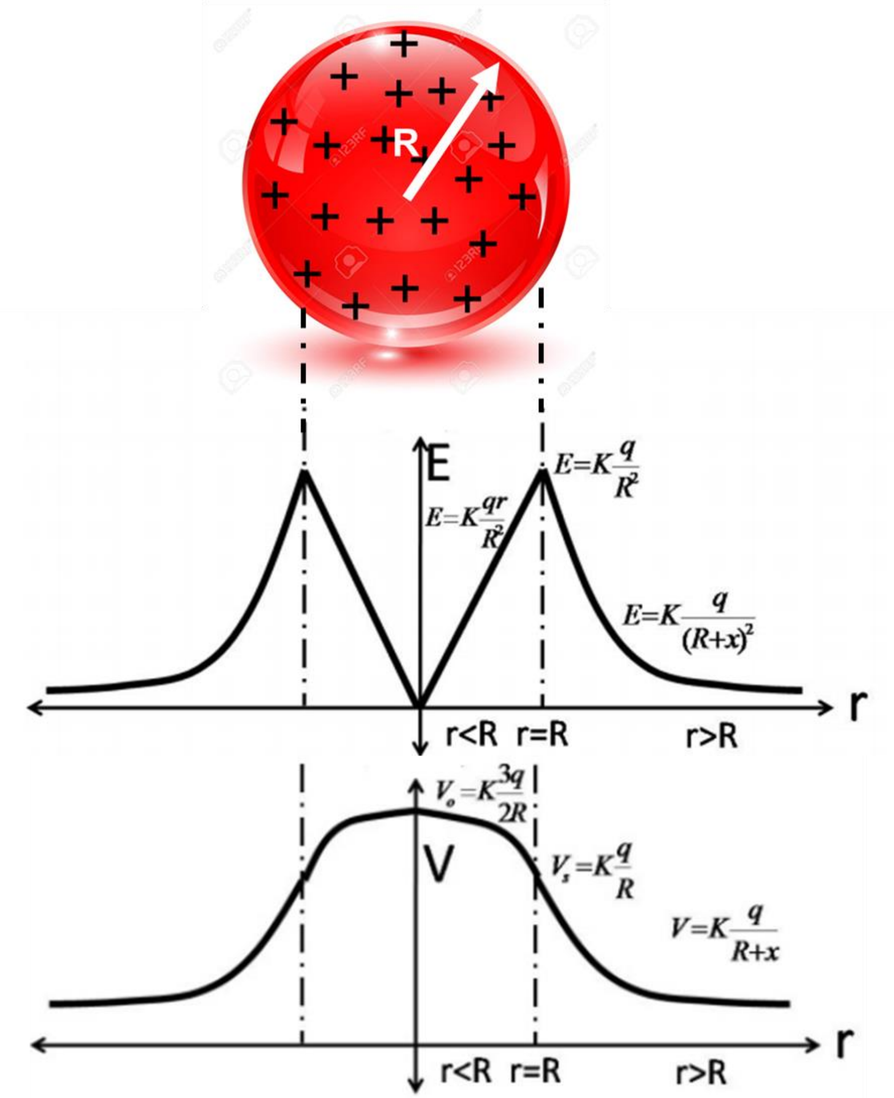


Potential difference of charged sphere

conducting



Non-conducting

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